Application of NLMS, PNLMS, and i-PNLMS Algorithms for NEXT Cancellation in xDSL Systems

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Abstract— **The application of the normalized least-mean square (NLMS), proportionate normalized least-mean square (PNLMS), and improved proportionate normalized least-mean square (i-PNLMS) algorithms for near-end crosstalk (NEXT) cancellation in digital subscriber line (xDSL) systems is examined. Extensive simulations have been carried out with these algorithms and on the basis of the results obtained the convergence rate and adaptation characteristics of the various algorithms are compared under different noise conditions. The simulations have been carried in two kinds of environment, namely, with just one type of DSL system and with a mixture of different DSL systems.**

I. Introduction

Digital subscriber line (DSL) technology exploits the existing copper loop plant to transfer data at Mb/s rates for high-speed Internet and other digital transmission services. In this technology, the DSL signal travels on a twisted pair loop from the central office to the subscriber. Many, sometimes thousands, of twisted pairs are bundled together in a single cable.

Telephone loops were designed and built to carry voice services, and at these low frequencies crosstalk is insignificant. However, at higher frequencies crosstalk becomes a major impairment. Considering the bundle as a multi-input/multi-output channel [1], the near-end crosstalk (NEXT) signals for a particular line can be cancelled by using an adaptive filter for each adjacent transmitted signal. This method can be very expensive if the bundle has a large number of twisted pairs.

Using various techniques [2][3], the computational complexity associated with NEXT cancellation can be significantly reduced. However, the computational complexity of the individual adaptive cancellers is still a very important issue especially in bundles with a large number of twisted pairs. At the same time, a high convergence rate in the adaptive canceller is equally desired as it helps to reduce the delay in cancelling the NEXT, which translates into a lower latency time for the system to achieve maximum data throughput.

The LMS algorithm [4] is a simple adaptation algorithm which is stable and works well even in noisy conditions. However, the convergence rate can be rather slow especially when the eigenvalue spread of the input signal is high. A variant of the LMS algorithm, the NLMS algorithm [5], has been shown to have a much better convergence rate with the advantage that it has almost the same computational complexity as the LMS algorithm. Due to these reasons, the NLMS algorithm is generally the algorithm of choice in NEXT cancellation. Recently, another variant of the LMS algorithm, the PNLMS algorithm was developed [6]. The idea behind this algorithm is to update each coefficient of the filter independently of the others by adjusting the adaptation step-size in proportion to the estimated filter coefficient. Unfortunately, when the impulse response is dispersive, the PNLMS algorithm converges more slowly than the NLMS algorithm. More recently an improved version of the PNLMS (i-PNLMS) algorithm was proposed [7], which promises to remove the disadvantages inherent in the PNLMS algorithm.

TABLE I
SUMMARY OF THE NLMS ALGORITHM

Parameters:
- M = number of taps
- μ = adaptation constant
- $0 < \mu < 2$
- a = positive constant
- $\mathbf{w}_i(n)$ = tap weight vector for ith filter

Data:
- $\mathbf{u}_i(n)$ = M-by-1 tap input vector for the ith filter
- $d(n)$ = desired response at time n

Computation:

$$e(n) = d(n) - \sum_i \mathbf{w}_i(n)\mathbf{u}_i(n)$$

$$\mathbf{w}_i(n+1) = \mathbf{w}_i(n) + \frac{\mu}{a+\|\mathbf{u}_i(n)\|^2}\mathbf{u}_i(n)e(n)$$

In this paper, the application of NLMS, PNLMS, and i-PNLMS algorithms for NEXT cancellation in xDSL systems is examined. Extensive simulations are carried out and on the basis of the results obtained the convergence rate and computational complexity of the various algorithms are compared under different noise conditions. In the simulations, two environments have been considered, namely, with just one type of DSL system and with a mixture of different DSL systems.

The paper is organized as follows. Section 2 describes the NLMS, PNLMS, and the i-PNLMS algorithms. In section 3 the advantages and disadvantages of the different algorithms in the context of NEXT cancellation in xDSL systems is discussed. In section 4, simulation results for the various algorithms pertaining to the convergence rate, are presented and discussed.

II. NLMS, PNLMS, and i-PNLMS Algorithms

In Table 1, the NLMS algorithm for cancelling multiple NEXT sources is shown. The normalized LMS algorithm

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TABLE II

SUMMARY OF THE PNLMS ALGORITHM

Parameters:
M = number of taps
μ = adaptation constant
$0 < \mu < 2$
δ = regularization parameter
δ_p = positive numbers (0.01 typical)
ρ = positive numbers ($5/M$ typical)
$\mathbf{w}_i(n)$ = tap weight vector for ith filter
Data:
$\mathbf{u}_i(n)$ = M-by-1 tap input vector for the ith filter
$d(n)$ = desired response at time n
Computation:
$e(n) = d(n) - \sum_i \mathbf{w}_i(n)\mathbf{u}_i(n)$
$\mathbf{w}_i(n+1) = \mathbf{w}_i(n) + \frac{\mu \mathbf{G}_i(n-1)\mathbf{u}_i(n)e(n)}{\mathbf{u}_i^t(n)\mathbf{G}_i(n-1)\mathbf{u}_i(n)+\delta}$
where
$\mathbf{G}_i(n-1) = \text{diag}\{g_{i0}(n-1), \cdots, g_{i(M-1)}(n-1)\}$
such that
$\gamma_{il}(n) = \max\{\rho\max[\delta_p, |w_{i0}(n)|, \cdots, |w_{i(M-1)}(n)|], |w_{il}(n)|\}$
and
$g_{il}(n) = \frac{\gamma_{il}(n)}{\sum_{l=0}^{M-1}\gamma_{il}(n)}, \quad 0 \le l \le M-1$

TABLE III

SUMMARY OF THE I-PNLMS ALGORITHM

Parameters:
M = number of taps
μ = adaptation constant
$0 < \mu < 2$
δ = regularization parameter
α = positive numbers (0 to -0.5 typical)
ε = small positive numbers
$\mathbf{w}_i(n)$ = tap weight vector for ith filter
Data:
$\mathbf{u}_i(n)$ = M-by-1 tap input vector for the ith filter
$d(n)$ = desired response at time n
Computation:
$e(n) = d(n) - \sum_i \mathbf{w}_i(n)\mathbf{u}_i(n)$
$\mathbf{w}_i(n+1) = \mathbf{w}_i(n) + \frac{\mu \mathbf{K}_i(n-1)\mathbf{u}_i(n)e(n)}{\mathbf{u}_i^t(n)\mathbf{K}_i(n-1)\mathbf{u}_i(n)+\delta}$
where
$\mathbf{K}_i(n-1) = \text{diag}\{k_{i0}(n-1), \cdots, k_{i(M-1)}(n-1)\}$
such that
$k_{il}(n) = \frac{1-\alpha}{2M} + (1+\alpha)\frac{|w_{il}(n)|}{2\|\mathbf{w}_i(n)\|_1+\varepsilon}$,
and
$0 \le l \le M-1$

exhibits a rate of convergence that is potentially faster than the standard LMS algorithm for both uncorrelated and correlated data [8]. Setting $\mu(n) = \frac{\mu}{\|\mathbf{u}_i(n)\|^2}$, the NLMS algorithm can be viewed as an LMS algorithm with a time-varying step-size parameter [5].

The NEXT cancellation algorithm using PNLMS algorithm is shown in Table 2. In this algorithm, an independent step size is assigned to each filter coefficient. The step sizes are calculated from the last estimate of the filter coefficient in such a way that a larger coefficient receives a larger increment, thus increasing the convergence rate of that coefficient. It has been shown that the PNLMS algorithm converges much faster than the NLMS algorithm when the impulse response is sparse [6].

Table 3 describes the NEXT cancellation algorithm using the i-PNLMS algorithm. The i-PNLMS algorithm promises to be as good as the PNLMS algorithm when the impulse response is sparse and equal to the NLMS algorithm when the response is highly dispersive. The basic difference with the PNLMS algorithm is the inclusion of another term in the estimation of the step size gain, i.e.,

$$k_{il}(n) = \frac{1-\alpha}{2M} + (1+\alpha)\frac{|w_{il}(n)|}{2\|\mathbf{w}_{il}(n)\|_1 + \varepsilon} \qquad (1)$$

Thus for $\alpha = 1$, the i-PNLMS and the NLMS algorithms are identical and for α close to 1, the i-PNLMS algorithm behaves like the PNLMS algorithm [7]. In the simulations for this paper, α was set to -0.3.

By assuming that the algorithm complexity is measured by computing the required number of multiplications, a simple LMS adaptive algorithm will have a complexity of order $2N$ for each iteration, or $2N$ multiplies. The NLMS algorithm requires an additional N multiplies to compute $\|\mathbf{u}_i(n)\|$. However, in practice NLMS algorithm is usually implemented as an order $2N$ algorithm simply by subtracting the square of the outgoing sample and adding in the square of the incoming sample in order to estimate $\|\mathbf{u}_i(n)\|$. If the same technique is applied to the PNLMS algorithm, then the complexity for PNLMS algorithm will be $3N$. The extra per-tap multiply is needed to multiply the gain distributors $g_{il}(n)$. In a similar manner, the complexity of the i-PNLMS algorithm is also of order $3N$.

III. SIMULATION RESULTS

Simulations were carried out to compare the performance of the various algorithms with respect to NEXT cancellation in xDSL systems. In the first simulation the crosstalk data was computed by simulating a bundle consisting of 7 twisted pairs of 26 AWG and length 7600 feet. HDSL2 was simulated on each of the pairs as specified in the T1E1.4 draft for the HDSL2 standard [9]. It was assumed that each of the

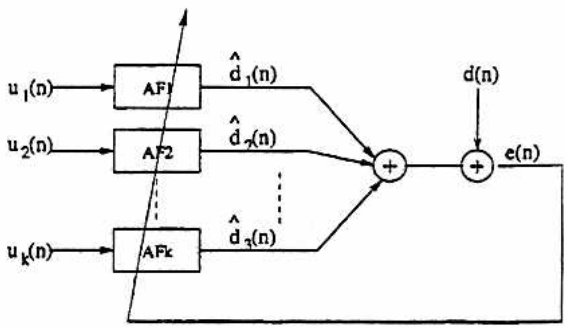

Fig. 1. Model of the NEXT cancellation system.

twisted pairs generates a crosstalk of certain magnitude on a particular line. The transmitted data symbols for all the lines are made available to each line for the purpose of crosstalk cancellation. Fig. 1 shows the block diagram of the model for NEXT cancellation. The filter length of each adaptive filter was set to 100. For the purpose of NEXT cancellation, the echo at the hybrid and the received signal have been removed

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